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ADDENDUM

Comment on K-H Yang's energy operator and gauge independent transition amplitudes

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Abstract. The purpose of this communication is to clarify and correct some of our remarks in a recent comment by us on Yang's energy operator.

In a recent publication (Feuchtwang *et al* 1984a) we discussed the 'gauge-independent formulation' (GIF) of Yang *et al* (1982a, b, 1983a, b). The purpose of this communication is to clarify and correct some of the remarks in our comment.

The classical Lagrangian of a system is unique only to within an arbitrary total time derivative of a function f of position and time; however, this does not affect the Euler-Lagrange equation of motion. A change in the gauge Λ of the vector potential is also equivalent to the addition of a total time derivative to the Lagrangian. Therefore, no directly observable physical quantity can depend either on the electromagnetic gauge $\Lambda(\boldsymbol{r}, t)$ or on the choice of the 'mechanical gauge' function $f(\boldsymbol{r}, t)$, and vice versa.

In classical physics, gauge convariance (or form invariance) is imposed on all relations between physical quantities in order to assure that the choice of gauges (Λ and f) has no physical consequence. This, however, imposes an explicit gauge dependence on some quantities (e.g. the canonical momentum) while others are identified as explicitly gauge independent (e.g. the velocity). Only the latter are directly observable. Similarly, in quantum physics, gauge covariance is imposed on all operators and operator relations such as equations of motion. However, in contrast with classical covariance, there is no unique definition of quantum gauge covariance. That is, the conventional statement of gauge covariance, that the change of gauges $A \to A'$ and $f \to f' = f + \Delta f$ induces on any arbitrary operator function F the transformation

$$F(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}; A_\mu, f) \to F(\hat{\boldsymbol{r}}, \hat{\boldsymbol{p}}; A'_\mu, f')$$

is not the only consistent definition of covariance (Feuchtwang *et al* 1984b). For instance, we can add an arbitrary gradient of a functon to $\boldsymbol{p} = (\hbar/\mathrm{i})\boldsymbol{\nabla}$ when performing a gauge transformation, rather than defining $\boldsymbol{p}$ to be gauge independent.

Furthermore, since only the expectation values of operators are observable, the gauge independence of an operator, derived from the imposition of covariance, does no longer guarantee the direct observability of the corresponding physical quantity. The latter only follows from the gauge independence of the expectation values. That is, the expectation values of the operator $\hat{O}$ will be observable if, and only if, the gauge-transformed operator $\hat{O}'$ satisfies the equation

$$\hat{O}' = U_{\Lambda+\Delta f} \hat{O} U^{-1}_{\Lambda+\Delta f}$$

where

$$U_{\Lambda+\Delta f} = \exp[(\mathrm{i}/\hbar)(e/c\Lambda + \Delta f)].$$

Thus, the connection of gauge covariance and direct observability is less immediate in quantum than in classical physics.

Yang *et al* (1982a, b, 1983a, b) suggested that the eigenstates $\{\psi_{\varepsilon_j(t)}\}$ of the time dependent 'mechanical' energy operator $\hat{\mathscr{H}}_{\mathrm{B}}$,

$$\hat{\mathscr{H}}_{\mathrm{B}} = \hat{\mathscr{H}} - eA_0 = \frac{1}{2m}\left(\hat{p} - \frac{e}{c}A\right)^2 + V(r)$$

constitute a uniquely suitable basis for the study of the interaction of material systems with radiation, within their 'gauge-independent formulation' of quantum mechanics. It is easily verified that the overlap coefficients

$$a_{\varepsilon_j(t)} = \langle \psi_{\varepsilon_j(t)} | \psi_A(t) \rangle$$

are independent of both the electromagnetic gauge $\Lambda(\boldsymbol{r}, t)$ and the mechanical gauge $f(\boldsymbol{r}, t)$. As noted by K-H Yang the latter point was missed by Feuchtwang *et al* (1984a). However, this gauge independence does not guarantee the physical significance of the $\{|a_{\varepsilon_j(t)}|^2\}$, as *observable and useful probabilities*, since the eigenvalues $\{\varepsilon_j(t)\}$ can have a rapid time dependence and require, in principle, 'instantaneous' measurements to monitor the transitions. This point is discussed in more detail by Feuchtwang *et al* (1984a).

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